Appendix A Descriptive Statistics and Data Visualization for Machine learning

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1. **Descriptive Statistics**
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    - Descriptive Statistics for Categorical Features
    - Populations & Samples
2. **Data Visualization**
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Descriptive Statistics

- The **arithmetic mean** (or **sample mean** or just **mean**) of a set of n values for a feature a, $a_1, a_2 \ldots a_n$, is denoted by the symbol $\overline{a}$, and is calculated as:

$$\overline{a} = \frac{1}{n} \sum_{i=1}^{n} a_i$$

Example

ID	1	2	3	4	5	6	7	8
Height	150	163	145	140	157	151	140	149

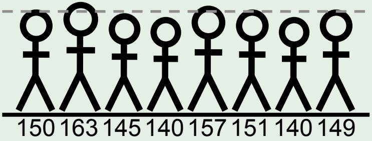


Figure: The members of a school basketball squad. The dashed grey line shows the arithmetic mean of the players' heights.

Example

ID	1	2	3	4	5	6	7	8
Height	150	163	145	140	157	151	140	149

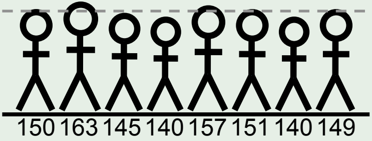


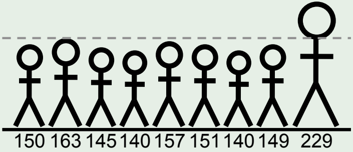
Figure: The members of a school basketball squad. The dashed grey line shows the arithmetic mean of the players' heights.

$$
\begin{aligned}
\overline{\textsc{Height}} &= \frac{1}{8} \times (150 + 163 + 145 + 140 + 157 + 151 + 140 + 149) \\
&= 149.375
\end{aligned}
$$

- The **arithmetic mean** is one measure of the **central tendency** of a **sample** (for our purposes a sample is just a set of values for a feature in an ABT).
- Any measure of **central tendency** is, however, just an approximation.

Example

- Suppose our basketball squad manage to sign a *ringer* measuring in at 229cm



- The arithmetic mean for the full group is 158.235cm and no longer represents the central tendency of the group.
- An unusually large or small value like this is referred to as an **outlier** - the arithmetic mean is very sensitive to outliers.

- The **median** of a set of values can be calculated by ordering the values from lowest to highest and selecting the middle value.
- If there is an even number of values in the sample then the median is obtained by calculating the arithmetic mean of the middle two values.

Example

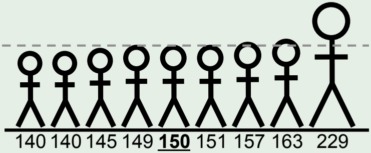


Figure: The members of the school basketball squad ordered by height, the dashed grey line shows the **median**.

ID	4	7	3	8	1	6	5	2	9
Height	140	140	145	149	**150**	151	157	163	229

- We also measure the **variation** in our data.
- In essence, most of statistics, and in turn analytics, is about describing and understanding variation.

Descriptive Statistics for Continuous Features

- The simplest measure of variation is the **range**:

$$\text{range} = max(a) - min(a)$$

Example

What is the range of the heights of the two basketball squads?

ID	1	2	3	4	5	6	7	8
Height	150	163	145	140	157	151	140	149

ID	1	2	3	4	5	6	7	8
Height	192	102	145	165	126	154	123	188

Example

What is the range of the heights of the two basketball squads?

$$\text{range} = 163 - 140 = 23$$

$$\text{range} = 192 - 102 = 90$$

Descriptive Statistics for Continuous Features

- The **variance** of a sample measures the average difference between each value in a sample and the mean of that sample.
- The **variance** of the n values of a feature a, $a_1, a_2 \ldots a_n$, is denoted $var(a)$ and is calculated as:

$$var(a) = \frac{\sum_{i=1}^{n} (a_i - \overline{a})^2}{n - 1}$$

Example

What is the variance of the heights of the two basketball squads?

ID	1	2	3	4	5	6	7	8
Height	150	163	145	140	157	151	140	149

ID	1	2	3	4	5	6	7	8
Height	192	102	145	165	126	154	123	188

Example

$$\begin{aligned} var(\textsc{Height}) &= \frac{(150-149.375)^2+(163-149.375)^2+\ldots+(149-149.375)^2}{8-1} \\ &= 63.125 \end{aligned}$$

$$\begin{aligned} var(\textsc{Height}) &= \frac{(192-149.375)^2+(102-149.375)^2+\ldots+(188-149.375)^2}{8-1} \\ &= 1,011.41071 \end{aligned}$$

- The **standard deviation**, *sd*, of a sample is calculated by taking the square root of the **variance** of the sample:

$$
\begin{aligned}
sd(a) &= \sqrt{var(a)} && (1)\\
&= \sqrt{\frac{\sum_{i=1}^{n}(a_i - \overline{a})^2}{n-1}} && (2)
\end{aligned}
$$

Example

What is the standard deviation of the heights of the two basketball squads?

ID	1	2	3	4	5	6	7	8
Height	150	163	145	140	157	151	140	149

ID	1	2	3	4	5	6	7	8
Height	192	102	145	165	126	154	123	188

Descriptive Statistics for Continuous Features

Example

$$
\begin{aligned}
sd(\textsc{Height}) &= \sqrt{63.125} \\
&= 7.9451\ldots
\end{aligned}
$$

$$
\begin{aligned}
sd(\textsc{Height}) &= \sqrt{1,011.41071} \\
&= 31.8026\ldots
\end{aligned}
$$

- **Percentiles** are another useful measure of the variation of the values for a feature: a proportion of $\frac{i}{100}$ of the values in a sample take values equal to or lower than the i^{th} percentile of that sample.

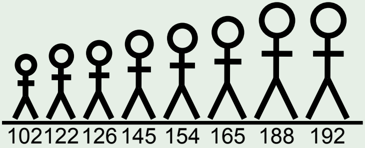
- To calculate the i^{th} percentile of the n values of a feature a, $a_1, a_2 \ldots a_n$:
  - First order the values in ascending order and then multiply n by $\frac{i}{100}$ to determine the *index*.
  - If the *index* is a whole number we take the value at that position in the ordered list of values as the i^{th} percentile.
  - If *index* is not a whole number then we **interpolate** the value for the i^{th} percentile as:

$$i^{th}\text{percentile} = (1 - index\_f) \times a_{index\_w} + index\_f \times a_{index\_w+1}$$

where *index_w* is the whole part of *index*, *index_f* is the fractional part of *index* and a_{index_w} is the value in the ordered list at position *index_w*.

Example

ID	2	7	5	3	6	4	8	1
Height	102	123	126	145	154	165	188	192



- What is the 25th percentile of the heights of the basketball squad?
- What is the 80th percentile of the heights of the basketball squad?

Example

- To calculate the 25^{th} percentile we first calculate *index* as $\frac{25}{100} \times 8 = 2$. So, the 25^{th} percentile is the second value in the ordered list which is 123.
- To calculate the 80^{th} percentile we first calculate *index* as $\frac{80}{100} \times 8 = 6.4$. Because *index* is not a whole number we set *index_w* to the whole part of *index*, 6, and *index_f* to the fractional part, 0.4. Then we can calculate the 80^{th} percentile as:

$$(1 - 0.4) \times 165 + 0.4 \times 188 = 174.2$$

- We can use percentiles to describe another measure of variation know as the **inter-quartile range**.
- The inter-quartile range is calculated as the difference between the 25th percentile and the 75th percentile.[1]

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[1] These percentiles are also known the **lower quartile** (or 1st quartile) and **upper quartile** (or 3rd quartile) hence the name inter-quartile range.

Example

For the heights of the first basketball team the inter-quartile range is $151 - 140 = 11$, while for the second team it is $165 - 123 = 42$.

Descriptive Statistics for Categorical Features

- For categorical features we are interested primarily in **frequency counts** and **proportions**.
  - The frequency count of each level of a categorical feature is calculated by counting the number of times that level appears in the sample.
  - The proportion for each level is calculated by dividing the frequency count for that level by the total sample size.
  - Frequencies and proportions are typically presented in a **frequency table**.
- The **mode** is a measure of the central tendency of a categorical feature and is simply the most frequent level.
- We often also calculate a **second mode** which is just the second most common level of a feature.

Table: A dataset showing the positions and weekly training expenses of a school basketball squad.

ID	Position	Training Expenses
1	center	56.75
2	guard	1,800.11
3	guard	1,341.03
4	forward	749.50
5	guard	1,150.00
6	forward	928.30
7	center	250.90
8	guard	806.15
9	guard	1,209.02
10	forward	405.72
11	center	550.00
12	center	223.89
13	center	103.23
14	forward	758.22
15	forward	430.79
16	forward	675.11
17	guard	1,657.20
18	guard	1,405.18
19	guard	760.51
20	forward	985.41

Descriptive Statistics for Categorical Features

Table: A frequency table for the POSITION feature from the professional basketball squad dataset in Table 4 [34].

Level	**Count**	**Proportion**
guard	8	40%
forward	7	35%
center	5	25%

- In statistics it is very important to understand the difference between a **population** and a **sample**.
- The term population is used in statistics to represent all possible measurements or outcomes that are of interest to us in a particular study or piece of analysis.
- The term sample refers to the subset of the population that is selected for analysis.
- The **margin of error** reported in poll results takes into account the fact that the result is based on a sample from a much larger population.

Table: A number of poll results from the run up to the 2012 US Presidential election.

Poll	**Obama**	**Romney**	**Other**	**Date**	**Margin of Error**	**Sample Size**
Pew Research	50	47	3	04-Nov	±2.2	2,709
Gallup	49	50	1	04-Nov	±2.0	2,700
ABC News/Wash Pos	50	47	3	04-Nov	±2.5	2,345
CNN/Opinion Research	49	49	2	04-Nov	±3.5	963
Pew Research	50	47	3	03-Nov	±2.2	2,709
ABC News/Wash Post	49	48	3	03-Nov	±2.5	2,069
ABC News/Wash Post	49	49	2	30-Oct	±3.0	1,288

Data Visualization

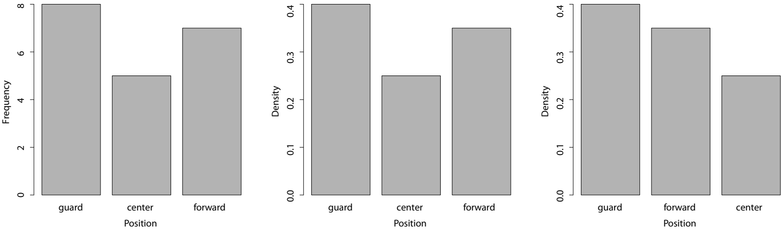
- When performing data exploration **data visualization** can help enormously.
- In this section we will describe three important data visualization techniques that can be used to visualize the values in a single feature:
  - the **bar plot**
  - the **histogram**
  - the **box plot**

Table: A dataset showing the positions and weekly training expenses of a school basketball squad.

ID	Position	Training Expenses	ID	Position	Training Expenses
1	center	56.75	11	center	550.00
2	guard	1,800.11	12	center	223.89
3	guard	1,341.03	13	center	103.23
4	forward	749.50	14	forward	758.22
5	guard	1,150.00	15	forward	430.79
6	forward	928.30	16	forward	675.11
7	center	250.90	17	guard	1,657.20
8	guard	806.15	18	guard	1,405.18
9	guard	1,209.02	19	guard	760.51
10	forward	405.72	20	forward	985.41

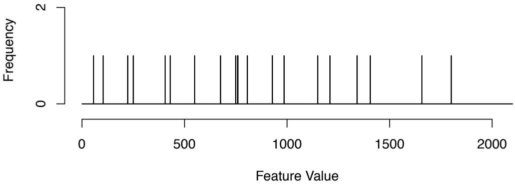
Bar Plots

Bar plots are great for categorical features



(a) Frequency (b) Proportion (c) Ordered

Bar plots don't work for continuous features



By dividing the range of a variable into intervals, or bins, we can generate **histograms**

(a) 200 unit intervals

Interval	**Count**	**Density**	**Prob**
$[0, 200)$	2	0.0005	0.1
$[200, 400)$	2	0.0005	0.1
$[400, 600)$	3	0.00075	0.15
$[600, 800)$	4	0.001	0.2
$[800, 1000)$	3	0.00075	0.15
$[1000, 1200)$	1	0.00025	0.05
$[1200, 1400)$	2	0.0005	0.1
$[1400, 1600)$	1	0.00025	0.05
$[1600, 1800)$	1	0.00025	0.05
$[1800, 2000)$	1	0.00025	0.02

(b) 500 unit intervals

Interval	**Count**	**Density**	**Prob**
$[0, 500)$	6	0.0006	0.3
$[500, 1000)$	8	0.0008	0.4
$[1000, 1500)$	4	0.0004	0.2
$[1500, 2000)$	2	0.0002	0.1

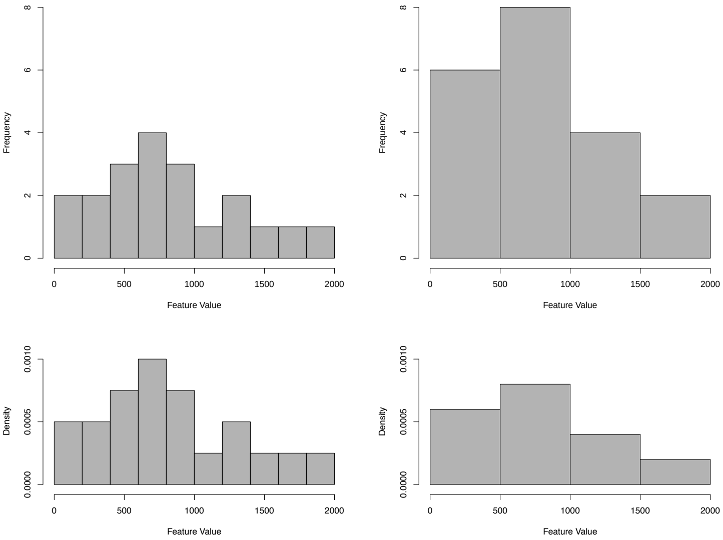


Figure: Frequency and density histograms for the continuous Training Expenses feature from Table 4 [34].

Box Plots

Box plots are another useful way of visualising continuous variables

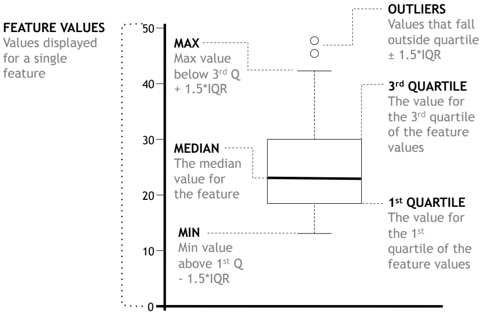

FEATURE VALUES
Values displayed for a single feature
50
40
30
20
10
0
MAX
Max value below 3rd Q + 1.5*IQR
MEDIAN
The median value for the feature
MIN
Min value above 1st Q - 1.5*IQR
OUTLIERS
Values that fall outside quartile ± 1.5*IQR
3rd QUARTILE
The value for the 3rd quartile of the feature values
1st QUARTILE
The value for the 1st quartile of the feature values

Figure: The structure of a box plot.

Box Plots

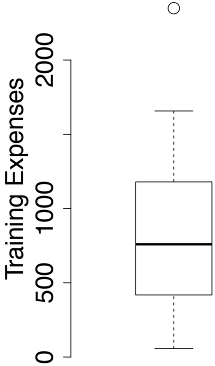


Figure: A box plot for the TRAINING EXPENSES feature from the basketball squad dataset in Table 4 [34].

Summary

1. **Descriptive Statistics**
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   - Descriptive Statistics for Categorical Features
   - Populations & Samples
2. **Data Visualization**
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   - Histograms
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3. **Summary**